Dependence Theory via Game Theory

A paper by Davide Grossi and Paolo Turrini

Presented by Rafik Hadfi

September 7th, 2010

Summary

1. Introduction
2. DependenceTheory
3. Game Theory recall
4. Dependencies in games
5. Dependency resolution
6. Conclusion

1. Introduction

- Social interaction analysis in Multi-agent systems.
- Possible approch for the analysis of Social interactions :

  Dependence Theory and/or Game Theory ?
- No research has been done relating these two approches.
- Unification of both theories :
  - Provides Dependence Theory with a mathematical foundation.
  - How Game Theory can incorporate dependence-theoretic aspects.

2. DependenceTheory 1/3

- is a body of **social science** theories predicated on the notion that resources flow from a "periphery" of poor and underdeveloped states to a "core" of wealthy states, enriching the latter at the expense of the former.

- It is a central contention of dependency theory that poor states are impoverished and rich ones enriched by the way poor states are integrated into the "world system".

- It is a theory of how developing and developed nations interact.

- It can be seen as an opposition theory to the popular free market theory of interaction, holding that open markets and free trade benefit developing nations, helping them eventually to join the global economy as equal players.

2. DependenceTheory 3/3

- Main premises of dependency theory :

  1. Poor nations provide natural resources, cheap labor, a destination for obsolete technology, and markets to the wealthy nations, without which the latter could not have the standard of living they enjoy.

  2. Wealthy nations actively perpetuate a state of dependence by various means. This influence may be multifaceted, involving economics, media control, politics, banking and finance, education, culture, sport, recruitment and training of workers and all aspects of human resource development .

  3. Wealthy nations actively counter attempts by dependent nations to resist their influences by means of economic sanctions and/or the use of military force.

- Dependency theory states that the poverty of the countries in the periphery is not because they are not integrated into the world system, or not 'fully' integrated as is often argued by free market economists, but because of how they are integrated into the system.

3. Game Theory recall 1/4

- Definition 1

A (strategic form) game is a tuple :

$$G=(N, S, \Sigma_i, \geq_i, o)$$

- N is a set of players,
- S is a set of outcomes,
- Σ_i is a set of pure strategies for player i in N,
- $\geq_i$ is total preorder over S,
- $o: \Sigma_1 \times .... \times \Sigma_N \rightarrow S$, is a bijective function from the set of strategy profiles $\Sigma=\prod\Sigma_{i..N}$ to S.

3. Game Theory recall 2/4

- A strategy profile is denoted σ,
- Players' strategies will be denoted as the ith projections of profiles : σ_i in Σ_i
- $\sigma_C = (\sigma_i)_{i \text{ in } C}$ denotes the strategy |C|-tuple of the set of agents C in N,
- Given a strategy profile σ and an agent i, the i-variant of σ is any pofile which differs from σ at most for σ_i : any profile (σ'_i, σ_{-i}) with σ' possibly different from σ and $-i = N\backslash\{i\}$.
- Similarily, the C-variant of σ is any pofile (σ'_C, σ_{-C}) with σ' possibly different from σ and $-C = N\backslash C$.
- Definition 2 (Equilibria).

  Let G be a game. A strategy profile σ is a :

  - BR-equilibrium (Nash Equilibrium) iff for all i, σ' :

    $o(\sigma) \geq_i o(\sigma'_i, \sigma_{-i})$

  - DS-equilibrium iff for all i, σ' : $o(\sigma_i, \sigma'_{-i}) \geq_i o(\sigma')$.

	Deny	Confess
Deny	2, 2	0, 3
Confess	3, 0	1, 1

Prisoner's dilemma

3. Game Theory recall 3/4

- A BR-equilibrium is a profile where all agents play a best response and a DS-equilibrium is a profile where all agents play a dominant strategy.
- Definition 3 (Coalitional Game).

A coalitional game is a tuple C = (N, S, E, ≥i)

where:

- N is a set of players,
- S is a set of outcomes,
- E is function $E : 2^N \rightarrow 2^{2^S}$
- ≥i is a total preorder on S.

The Effectivity function E assigns to every coalition the sets of states that the coalition is able to enforce.

3. Game Theory recall 4/4

- Definition 4 (The Core).

Let **C** = (N, S, E, ≥i) be a coalitional game. We say that a state *s* is dominated in **C** if for some C and X in E(C), it holds that $x >_i s$ for all x in X, i in C.

The core of **C**, in symbols *CORE(**C**)* is the set of undominated states.

→ The core is the set of those states in the game that are stable, *i.e.* for which there is no coalition that is at the same time able and interested to deviate from them.

4. Dependencies in games

- Study of the dependence from a Game Theoretic perspective.
  - → *" i <u>depends</u> on j <u>for achieving goal</u> g "*

- Dependence is represented as a need for a favor :

“x depends on y with regard to an act useful for realizing a state p when p is a goal of x’s and x is unable to realize p while y is able to do so.”

- Hence, from a game theoretic perspective, this formal relation means :

A player i depends on a player j for the realization of a state p, *i.e.* of the strategy profile such that $o(\sigma) = p$, when, in order for to occur, j has to favour i, that is, it has to play in i’s interest.

→ i depends on j for σ when, in order to achieve σ, j has to do a favour to i by playing σ_j (which is obviously not under i’s control)

4. Dependencies in games

- Definition 5 (Best For Someone Else).

Assume a game $G=(N, S, \Sigma i, \geq i, o)$ and let i,j in N.

1) Player j's strategy in σ is a best response for i iff for all σ', $o(\sigma) \geq_i o(\sigma'_j, \sigma_{-j})$

2) Player j's strategy in σ is a dominant strategy for i iff for all σ', $o(\sigma_j, \sigma'_{-j}) \geq_i o(\sigma')$

→ Generalization of the standard definitions of best response and dominant strategy by allowing the player holding the preference to be different from the player whose strategies are considered. By setting $i = j$ we obtain the usual definitions.

4. Dependencies in games

- We can mathematically define the notion(s) of dependence as game theoretic notions :
- Definition 6 (Dependence).

Let G=(N, S, Σi, ≥i, o) be a game and i,j in N.

1) Player i BR-depends on j for strategy σ iff σ_j is a best response for i in σ.

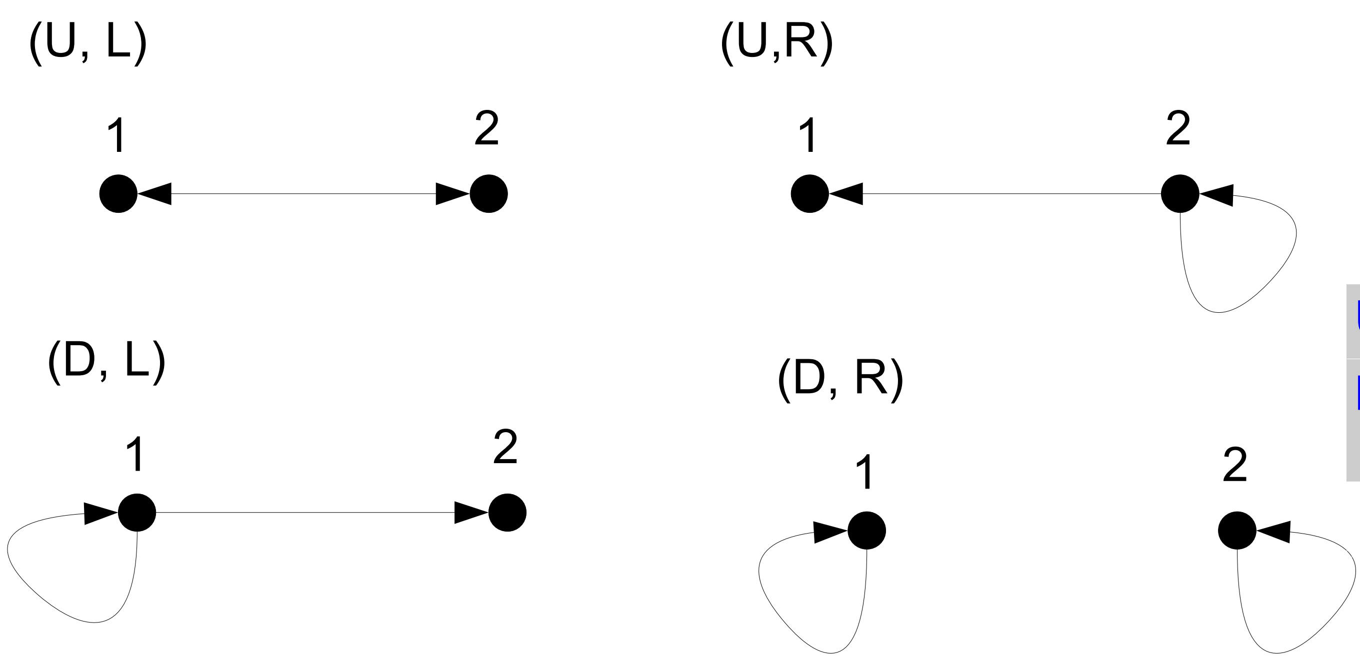
2) Player i DS-depends on j for strategy σ iff σ_j is a dominant strategy for i.

→ i depends on j for profile σ in a best response sense if, in σ, j plays a strategy which is a best response for i given the strategies in σ_{-j} (and hence given the choice of i itself).

4. Dependencies in games

- Therefore, with any game G two dependence structures can be defined, based on the notions of best response and dominant strategy:
  - (N, $R_{BR,\sigma}$)
  - (N, $R_{DS,\sigma}$), where σ in $\Sigma_1 \times .... \times \Sigma_N$.

- Graphical representation of the BR-dependences (Prisoner's dilemma)

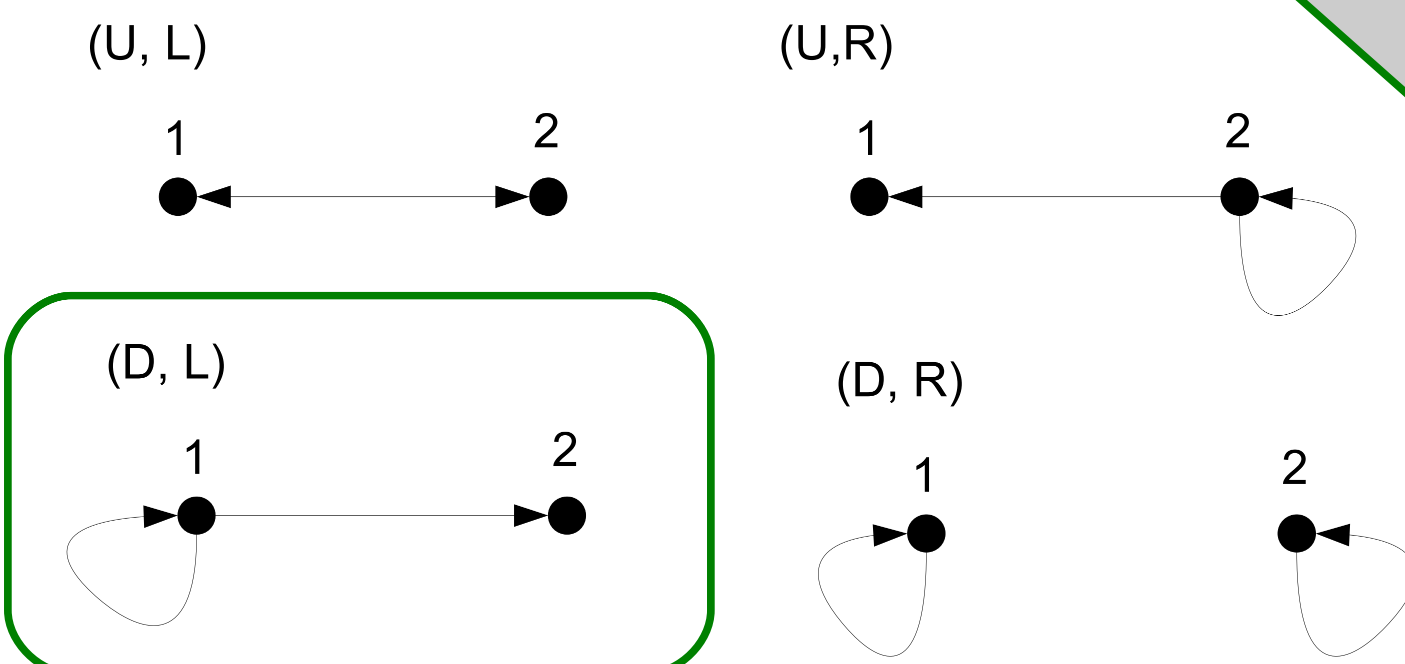


	L -Deny	R -Confess
U -Deny	2, 2	0, 3
D -Confess	3, 0	1, 1

Prisoner's dilemma

4. Dependencies in games

If this outcome were to be achieved, **Row** would depend on **Column** in that, while **Row** plays its **dominant** strategy, **Column** has to play a **dominated** one which maximizes **Row**'s welfare.



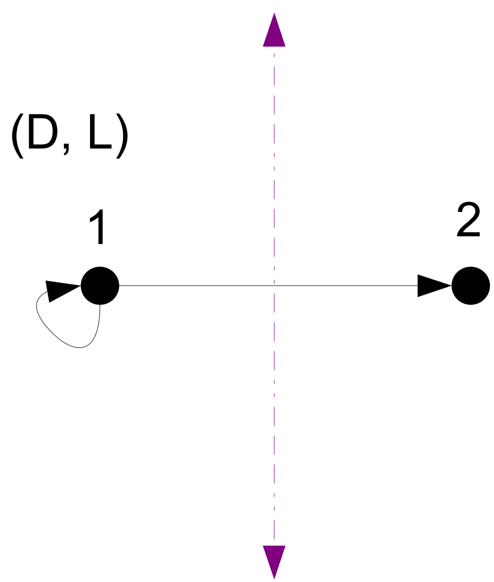
	L -Deny	R -Confess
U -Deny	2, 2	0, 3
D -Confess	3, 0	1, 1

Prisoner's dilemma

4. Dependencies in games

Cycles

- Difference between the pair of profiles (D,R) and (U,L) and the pair of profiles (D,L) and (U,R) in term of symmetries.
- (D, L) Row **BR-depends** on Column while Column does not **BR-depend** on Row.
- Game-theoretic **instability** of the profiles : lack of **balance** or **reciprocity** in the dependence structrure (Graph).



Symmetry **Asymmetry**

	L	R
U	2, 2	0, 3
D	3, 0	1, 1

4. Dependencies in games

Cycles

- A dependence is reciprocal when it allows for the occurence of a *"Social exchanche"*, or exchange of favours between two involved agents.

➔ It happens in the presence of cycles in the dependence relation.

- Definition 7 (Dependence Cycles)

Let $G=(N, S, \Sigma_i, \geq_i, o)$ be a game and i,j in N. $(N, R_{x,\sigma})$ be its depence structure for profile σ with x in {BR, DS}.

An $R_{x,\sigma}$-dependence cycle *c* of length k-1 in G is a tuple $(a_1, ..., a_k)$ such that:

$a_1, ..., a_k$ in N; $a_1=a_k$; for all a_i,a_j with $1\leq i \neq j \leq k$; $a_i \neq a_j$;

$a_1\ R_{x,\sigma}\ a_2\ R_{x,\sigma}\ a_3\ R_{x,\sigma}\ a_4\ R_{x,\sigma},.......,a_{k-2}\ R_{x,\sigma}\ a_{k-1}\ R_{x,\sigma}\ a_k$

Given a cycle $c=(a_1, ..., a_k)$, its orbit $O(c)=\{a_1, ..., a_{k-1}\}$ denotes the set of it elements.

4. Dependencies in games

Cycles

- A dependence is reciprocal w[...] *exchanche"*, or exchange of f[...]

➔ It happens in the presence of[...]

➔ Cycles are sequences of pairwise different agents, except for the first and the last which are equal.

➔ All agents are linked by a dependence relation.

- Definition 7 (Dependence Cycles)

Let G=(N, S, Σi, ≥i, o) be a game and i,j in N. (N, $R_{x,\sigma}$) be its depence structure for profile σ with x in {BR, DS}.

An $R_{x,\sigma}$-dependence cycle *c* of length k-1 in G is a tuple (a_1, ..., a_k) such that:

$a_1, \ldots, a_k$ in N; $a_1=a_k$; for all a_i,a_j with $1\leq i \neq j \leq k$; $a_i \neq a_j$;

$a_1 \; R_{x,\sigma} \; a_2 \; R_{x,\sigma} \; a_3 \; R_{x,\sigma} \; a_4 \; R_{x,\sigma}, \ldots\ldots, a_{k-2} \; R_{x,\sigma} \; a_{k-1} \; R_{x,\sigma} \; a_k$

Given a cycle *c*=(a_1, ..., a_k), its orbit O(*c*)={a_1, ..., a_{k-1} } denotes the set of it elements.

4. Dependencies in games

- Cycles as equlibria somewhere else

Considering the game $G=(N, S, \Sigma i, \geq i, o)$ and a bijection $\mu:N\rightarrow N$.

The μ-permutation of the game G is the game $G_\mu=(N, S, \Sigma_\mu, \geq i, o_\mu)$ where for all i in N, $\Sigma_{i,\mu}=\Sigma_{\mu(i)}$ and the outcome function $o: \Sigma_{\mu(1)} \times .... \times \Sigma_{\mu(N)} \rightarrow S$, is such that

$o_\mu(\mu(\sigma))=o(\sigma)$ with $\mu(\sigma)$ denoting the permutation of σ according to μ.

➔ Example : Two Horsemen.

- Theorem (Reciprocity in equilibrium).

  Let G be a game and $(N, R_{\sigma,x})$ be its dependence structure with x in {BR,DS} and σ be a profile.

  It holds that σ is *x-reciprocal* iff there exists a bijection $\mu:N\rightarrow N$, σ is a x-equilibrium in G_μ

5. Dependency resolution : Agreements

- Definition 8 (Agreements)

  Let G be a game and (N, $R_{\sigma,x}$) be its dependence structure in σ with x in {BR,DS}, and let i,j in N.

  A pair (σ, μ) is an **x-agreement** for G if σ is an **x-reciprocal** profile, and μ a bijection which **x-implements** σ in G.

  The set of x-agreements of a game G is denoted x-ARG(G)

- An agreement, (BR or DS type), can be seen as the result of agents' coordination selecting a desirable outcome and realizing it by an appropriate exchange of strategies.
- A bijection μ formalizes a precise idea of social exchange in a game-theoretic setting.

6. Conclusion

- Central notions of Dependence Theoretic such as the notion of cycle can be characterized from a game theoretic perspective.

- Dependence theory gives new types of cooperative games where solution concepts as the core can be applied to obtain an 'analytical predictive power' that dependence theory unsuccessfully looked for since it begining.

References

- Shoham
  - Mixed strategy, Def 3.2.4 (p59)
  - Best response, Def 3.3.3 (p61)
  - Nash Equilibrium, Def 3.3.4 (p62)
  - Domination, Def 3.4.8-9 (p77)
- LAMBERSON, p6-7