Cell Groups Reveal Structure of Stimulus Space

Carina Curto and Vladimir Itskov
PLoS Comput Biol 4.10 (2008)

Presented by Rafik Hadfi

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and a brief look at Persistent Homology

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Summary

- Introduction
- Stimulus Construction

- Main Results Of the Paper
- Results
  1. Cell Groups Reveal Place Field Intersection Information
  2. Global topological features
  3. Topological Features Can Be Extracted from Cell Groups
  4. An Internal Representation of Space Can Be Built from Cell Groups
- Conclusion and ideas

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Introduction

- An important task of the brain is to represent the outside world
  - How does it do it when it can only rely on neural responses ?
  - What can be learned about a space of stimuli using only the action potentials (spikes) of cells ?

Stimulus Construction

- Three steps:
  1. Characterizing the space of (relevant) stimuli
  2. Constructing functions relating stimuli to neuronal responses
  3. Use the functions, together with new neuronal activity, to decode new stimuli

- Example: In the case of hippocampal place cells, the space of stimuli could be the animal current spatial environment; for every place cell one computes a place field, i.e., a function that assigns a firing rate to each position in space.

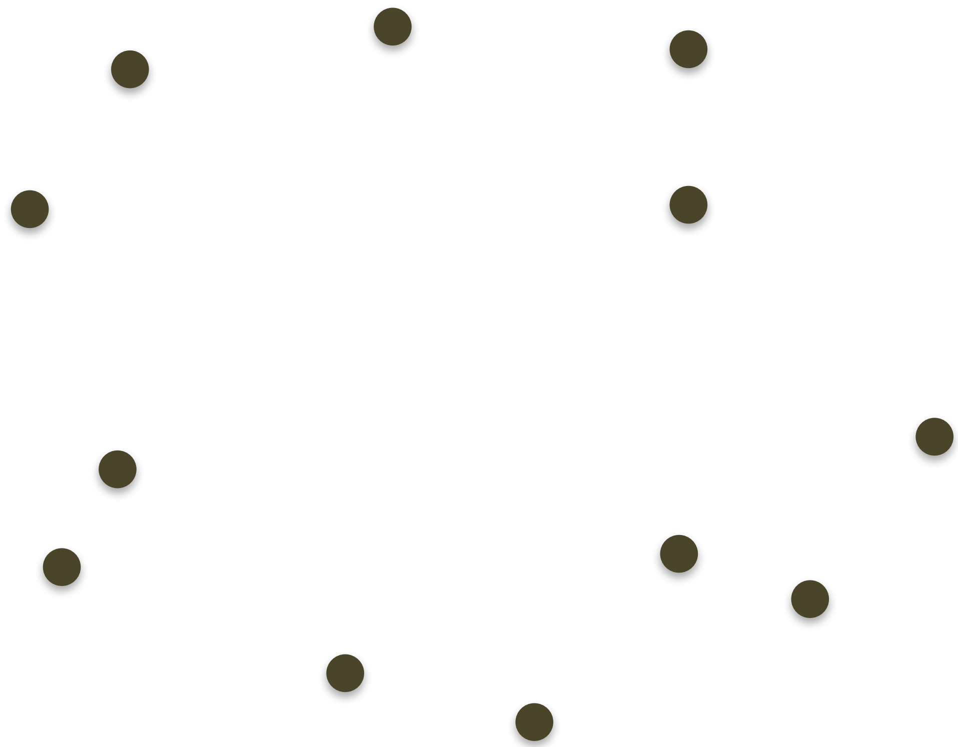
Main Results Of the Paper

- Using hippocampal place cells as a model system, one can
  1. Extract global topological features of the environment and
  2. Reconstruct an accurate geometric representation of physical space, up to an overall scale factor, that can be used to track the animal's position

- How? Using standard tools from algebraic topology (persistent homology) and graph theory

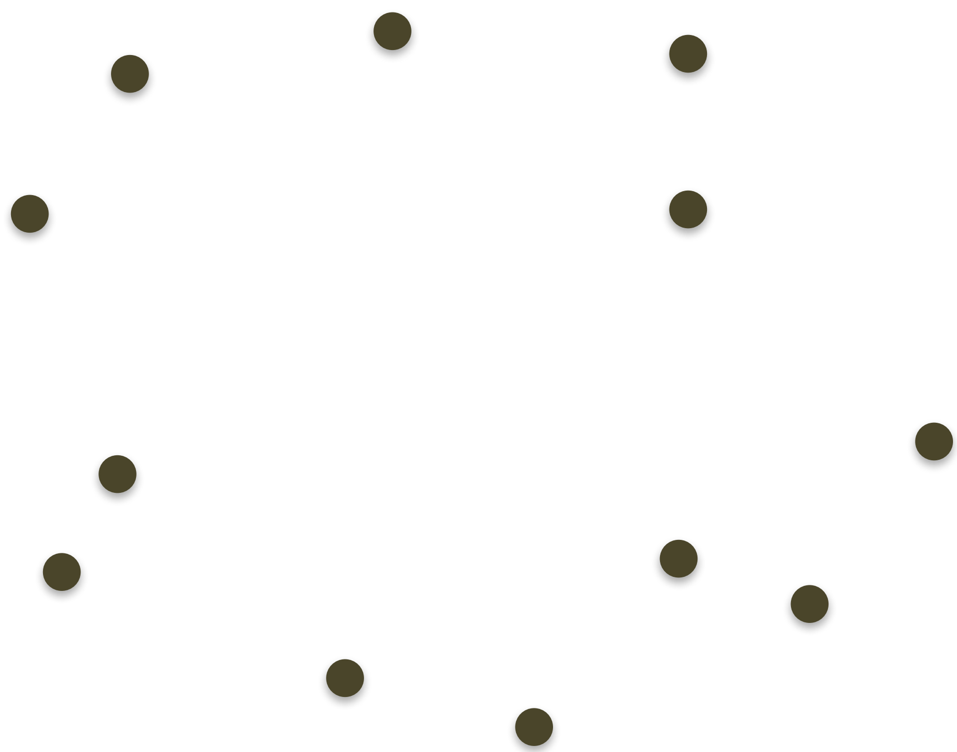
  - Neither place fields, nor precise spike timing, nor any prior independent measurements of position are needed

Persistent Homology



Persistent Homology

- What topological features does the following data exhibit?



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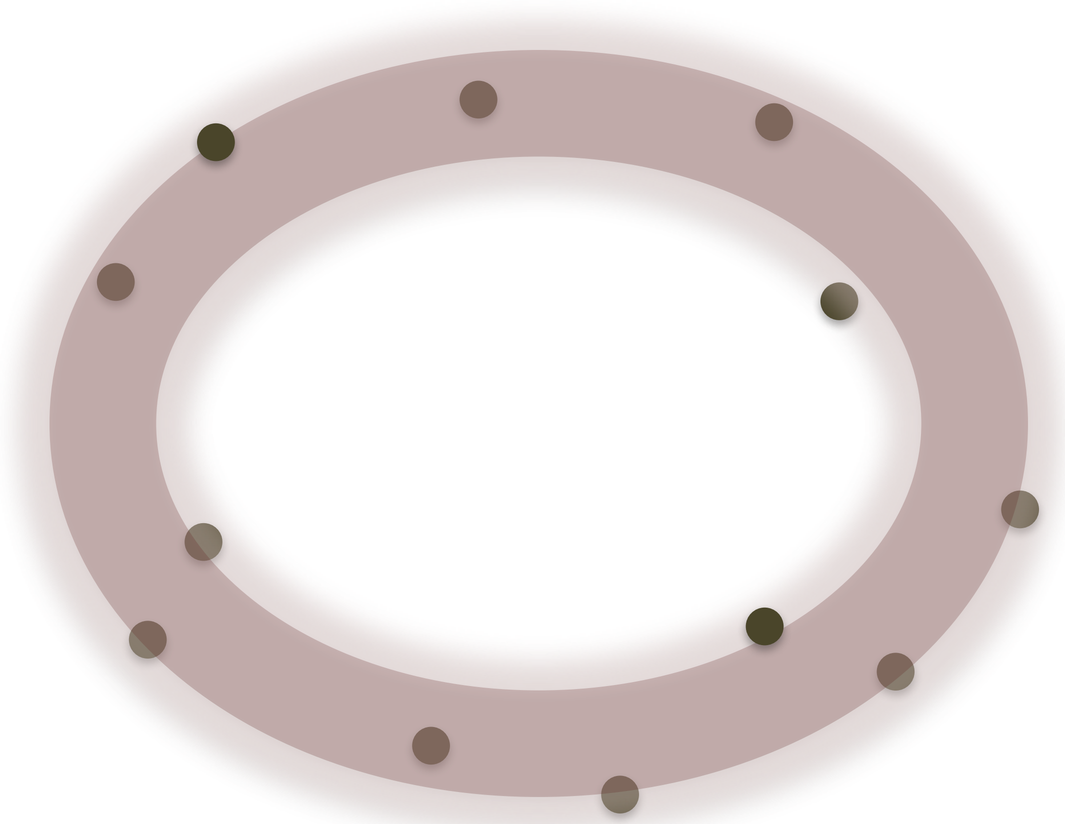

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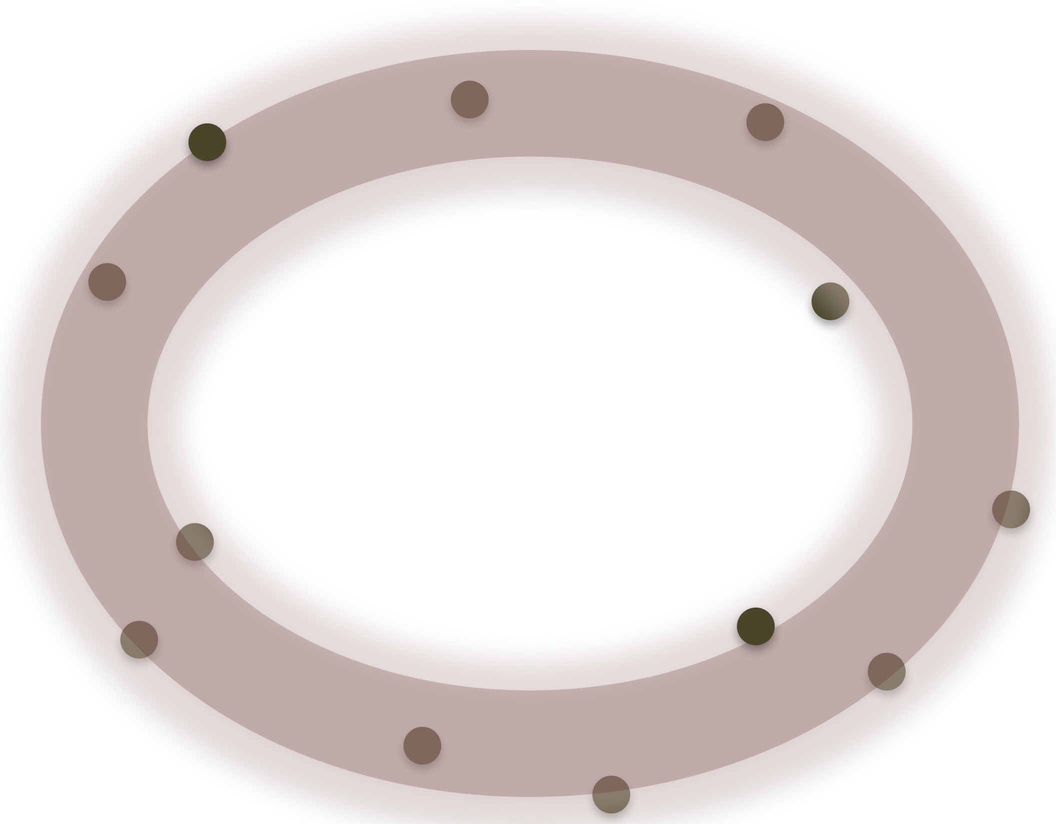


But: Discrete points have trivial topologies!

Persistent Homology

- Persistent homology is an algebraic method for discerning topological features of data

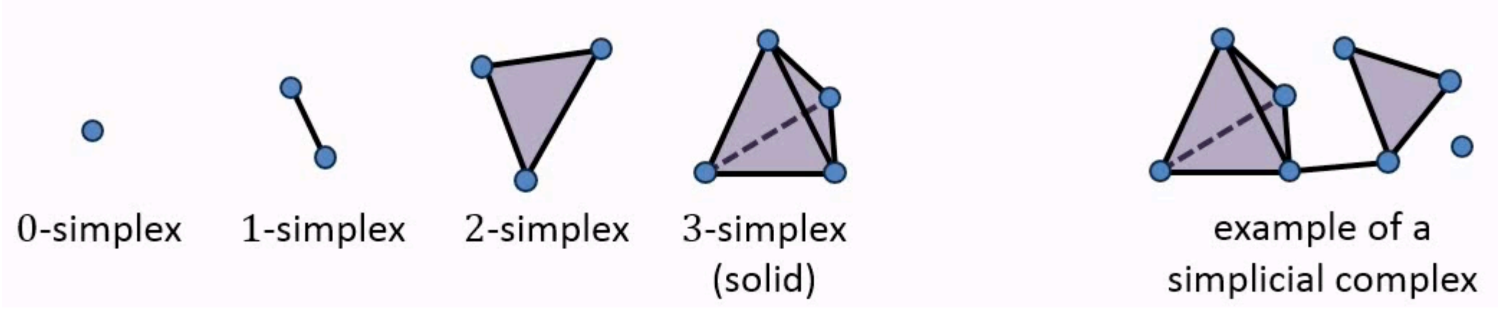
Components, holes, voids, graph structures

Set of discrete points with a metric

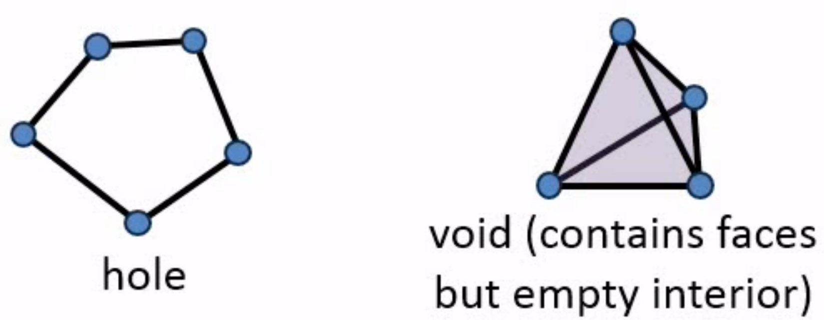
- A method for computing topological features of a space at different spatial resolutions

Persistent Homology

- To find the persistent homology of a space, the space must first be represented as a simplicial complex
  - A simplicial complex is built from points, lines segments, triangular faces, and their n-dimensional counterparts



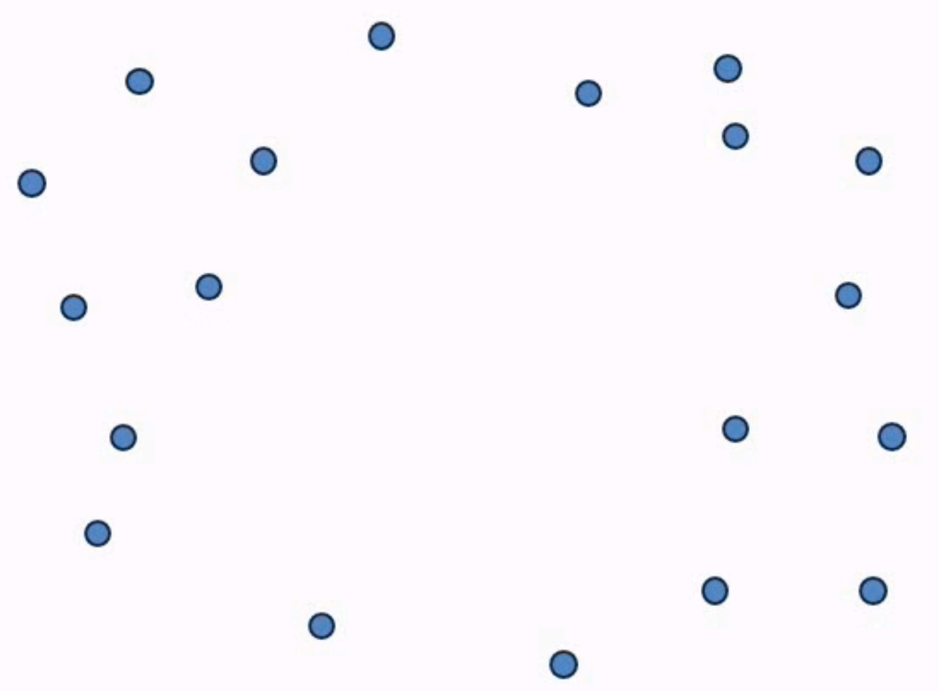
- Homology counts components, holes, voids, etc.



Persistent Homology

- Start by connecting the nearby points

1. Choose a distance d



Persistent Homology

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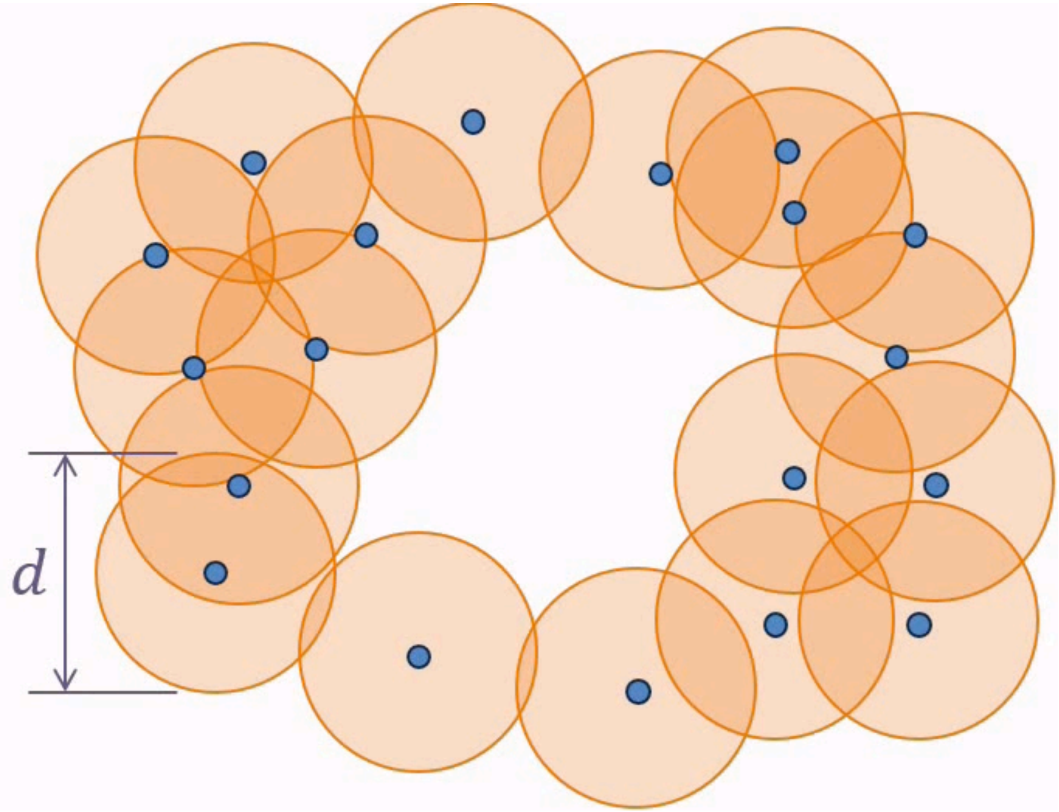
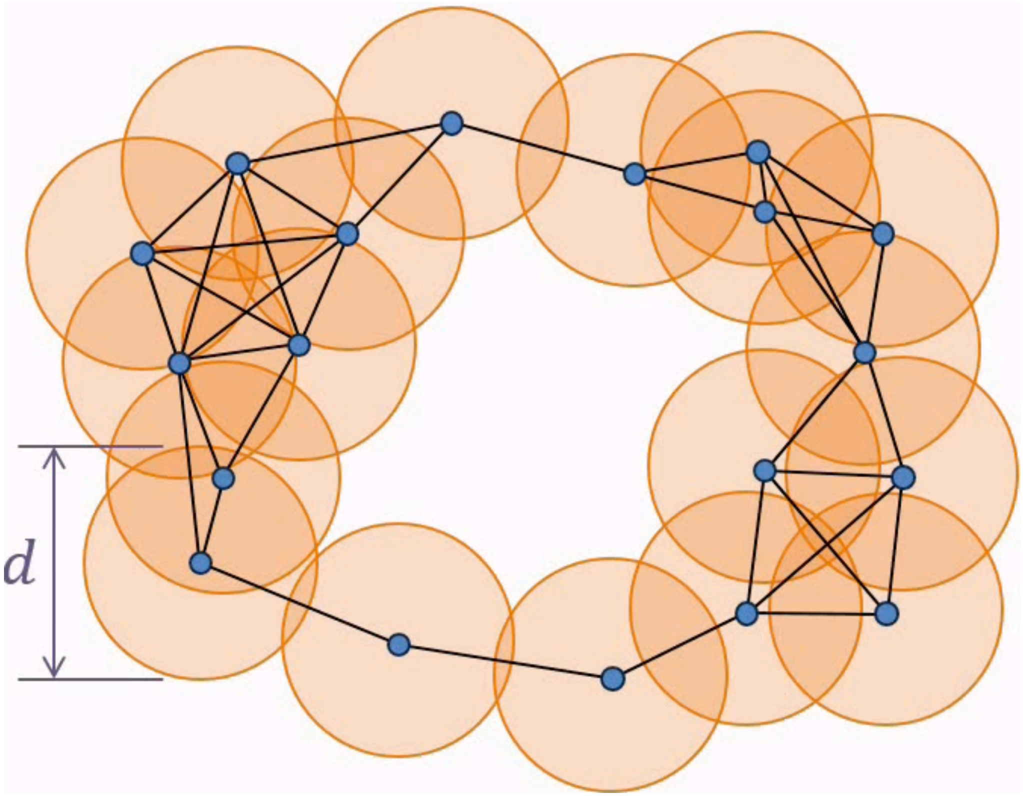


Persistent Homology

- Start by connecting the nearby points

1. Choose a distance d
2. Connect points (A,B) with distance(A,B)<d



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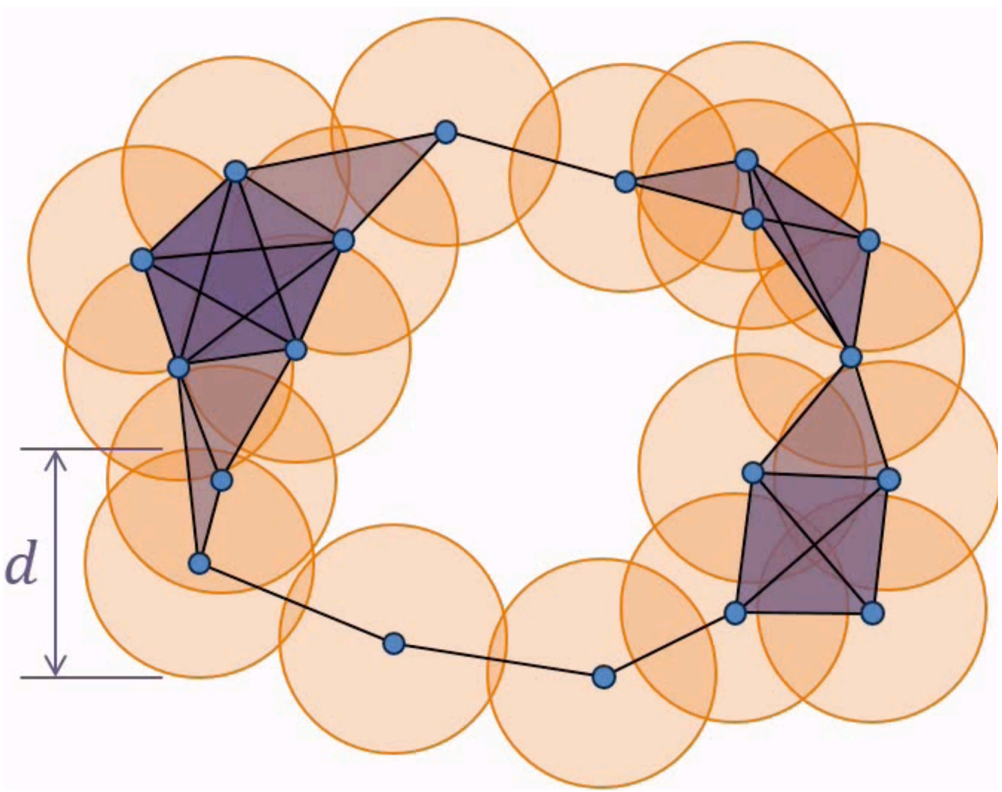

Persistent Homology

- Start by connecting the nearby points

1. Choose a distance d
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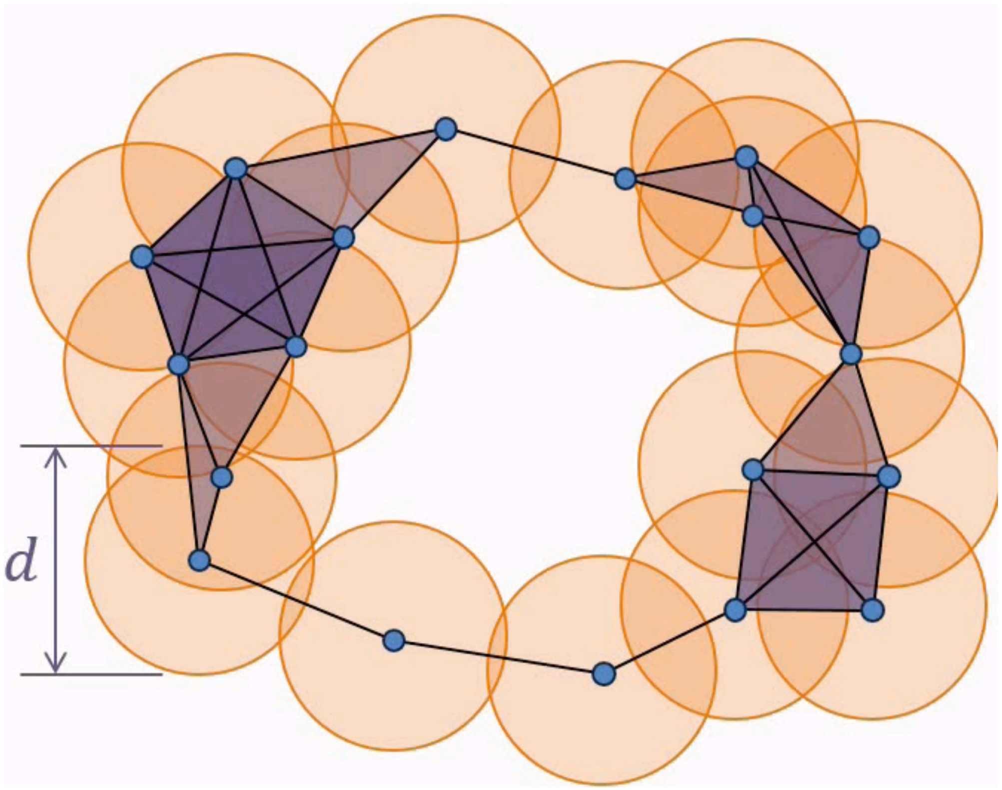
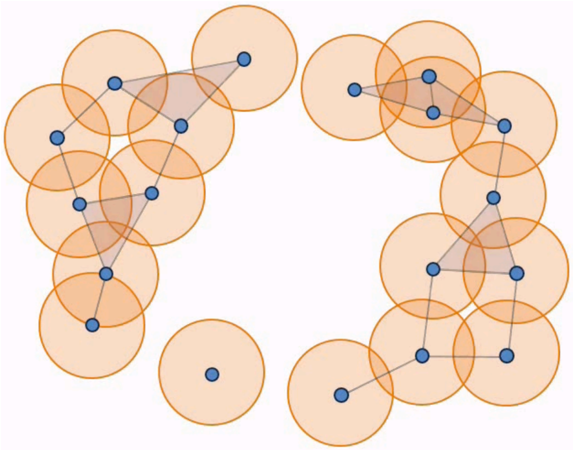
3. Fill in complete simplices
4. Homology detects the holes



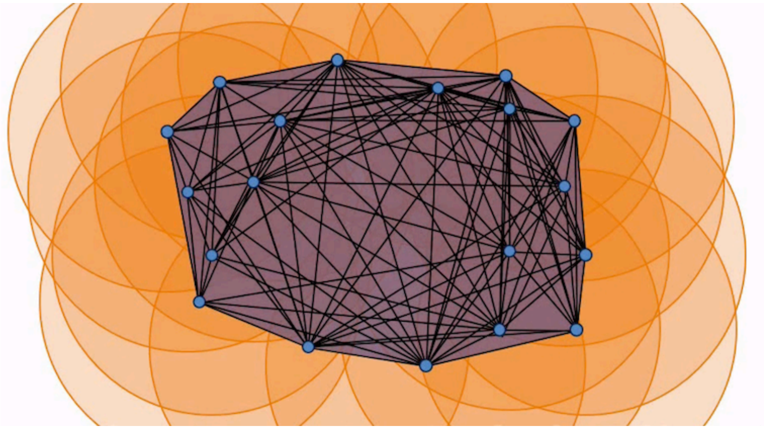
How do we choose d?

Persistent Homology

- If d is too small, we detect noise

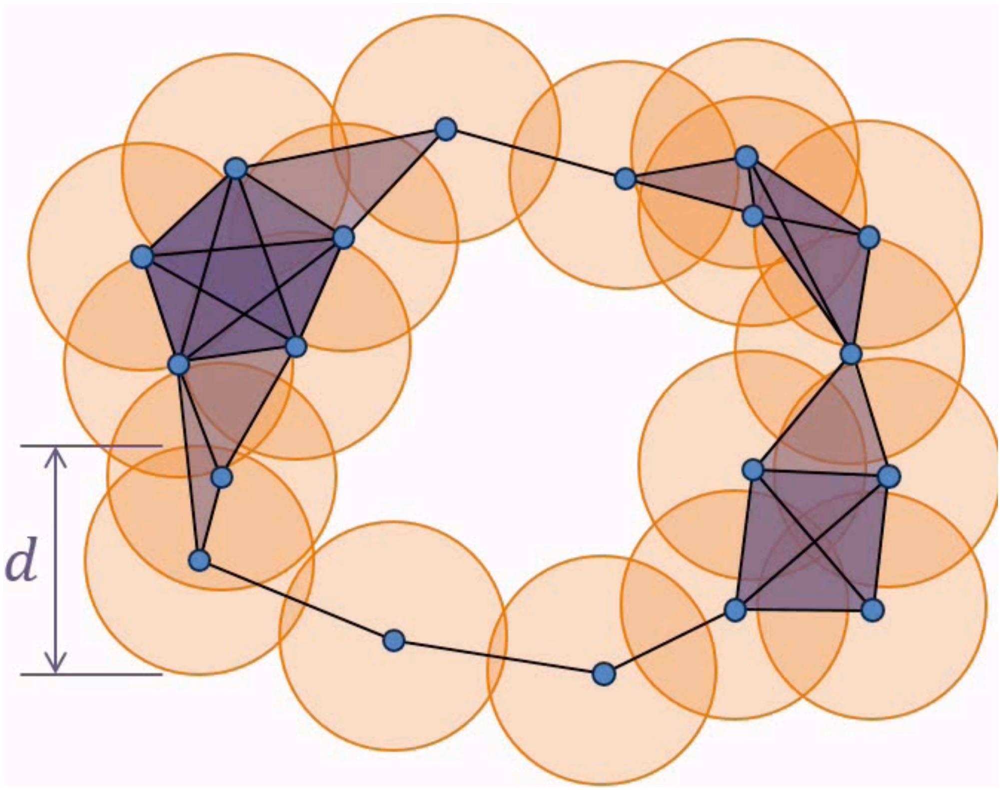


- If d is too large, we get one giant simplex (the trivial homology)



Persistent Homology

- Solution: consider all distances, until finding the appropriate one
- Each hole appears at a particular value of d and disappears at another value



(1) Cell Groups Reveal Place Field Intersection Information

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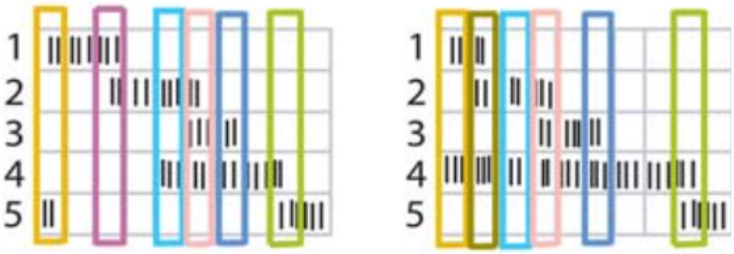

(A) Sample rasters for the population activity of five place cells in two different Environments. Cell groups are obtained by identifying subsets of cells that co-fire within a coarse time window

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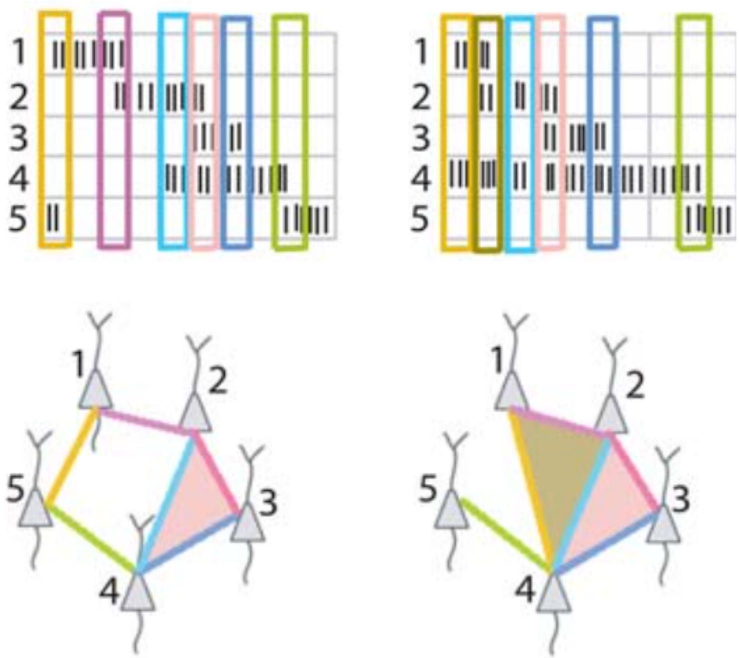

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(B) Two examples of five-cell configurations depicting collections of cell groups obtained from the sample rasters in (A). An edge represents a cell group with two cells and a shaded triangle indicates a cell group with three cells (A)

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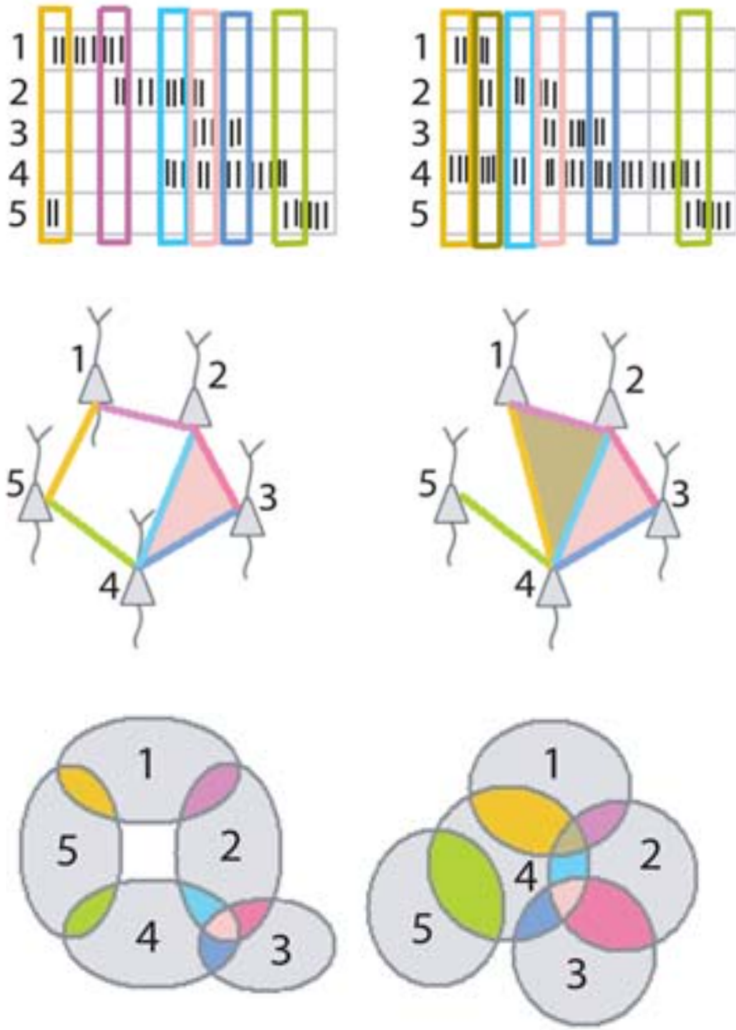

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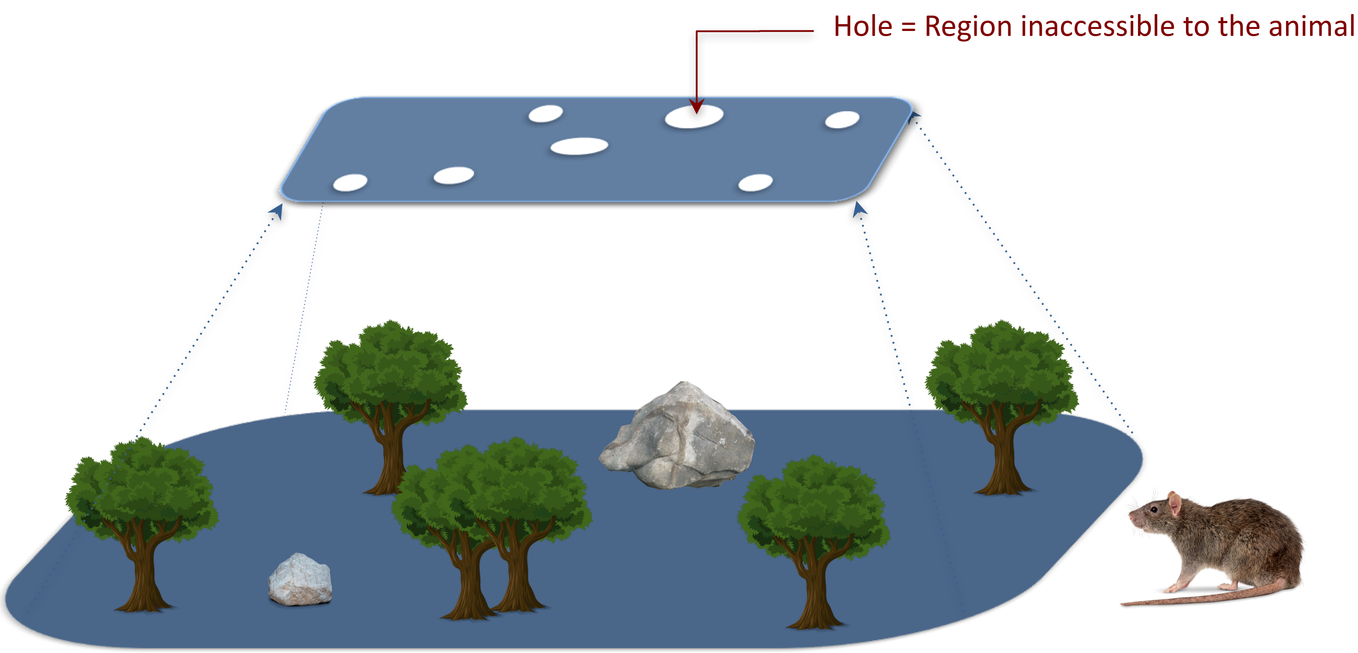
(C) Cells that co-fire have overlapping place fields. Each cell group in (A), (B) corresponds to a particular intersection of place fields, denoted with matching color. The place field intersection pattern fully determines the topology of a space covered by convex place fields

(2) Global topological features

- Often times an animal's physical space has ''holes''—i.e., regions in the interior of the environment where the animal is unable to go.
- Example: a rat may be confined to a platform with one or more holes in the middle; similarly, there may be large objects inside the environment providing obstructions to the animal's path.



(3) Topological Features Can Be Extracted from Cell Groups

- Computing the homology groups of the environment

- Steps:
  1. Population of place cells
  2. Collection of cell groups
  3. Intersection information
  4. Homology groups

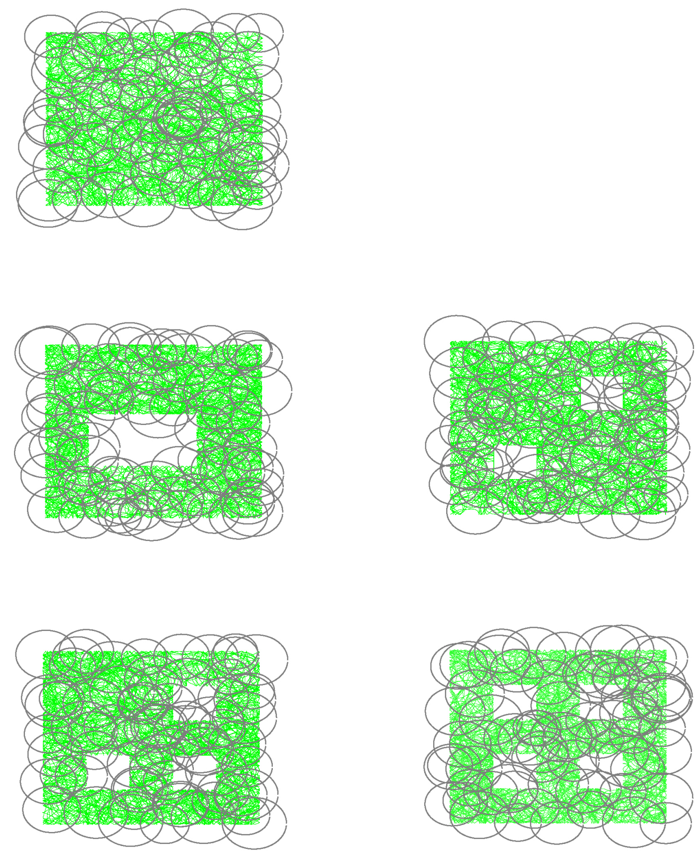
(3) Topological Features Can Be Extracted from Cell Groups

- Random-walk trajectories, 5 x 2-D flat environments, each of side length L=1 m, with N = 0,1,...,4 holes

- In each of 300 trials, each of the five environments was covered by 70 single-peaked place fields with varying radii (0.1–0.15 L) and randomly-chosen centers

- For each trial, the first five homology groups (H0,...,H4) were computed

- A trial was deemed to be correct if and only if all homology groups matched the topology of the underlying space, and incorrect if at least one homology group did not match.



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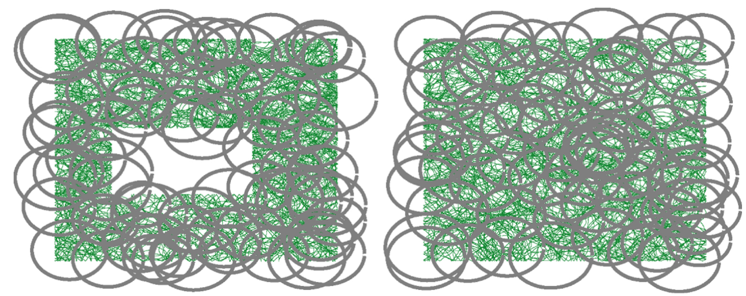

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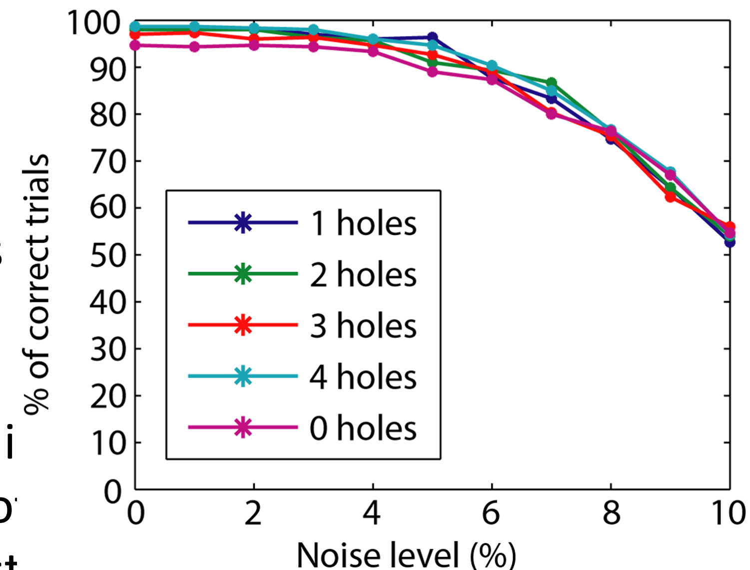

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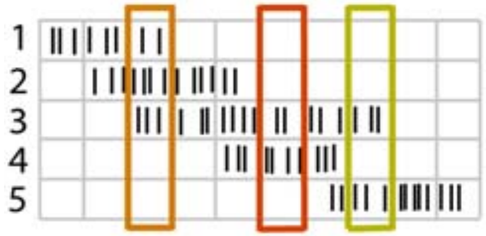
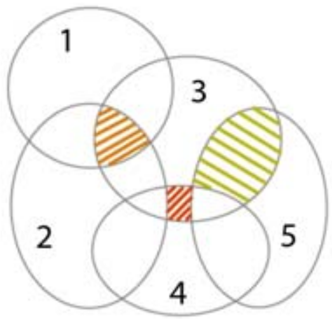
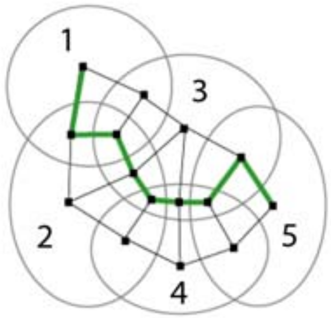


(A) Example spike trains from five place cells. Each time bin (columns) represents two theta cycles



(B) Place field intersection pattern derived from cell groups in (A). Shaded regions correspond to cell groups inside rectangles of the same color in (A)



(C) The pattern of intersections represented by a graph, with vertices (■) for each cell group, and edges connecting neighbors. A trajectory is inferred from the example data, by ‘‘connecting the dots’’ to match the sequence of cell groups in (A)

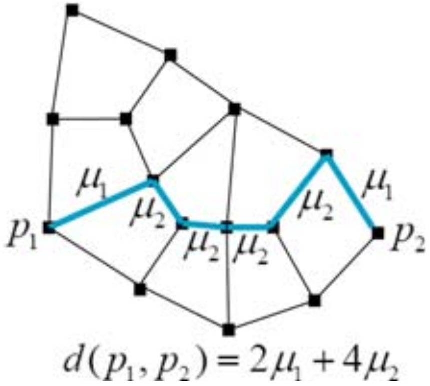
(4) An Internal Representation of Space Can Be Built from Cell Groups

- Define a dissimilarity index µk on neighboring cell groups as the average relative distance between the centers of adjacent regions with overlap degree k, assuming place fields of equal radius

- µk should be derivable from basic geometry, as it depends only on general and unchanging properties of physical space

- The distance between any two cell groups (two vertices) in the graph can then be defined as the length of a shortest path between those points.

→ Natural metric on cell groups.
→ Internal representation of the external space.



(D)

Conclusion

- Using standard tools from algebraic topology (persistent homology) and graph theory, one could:
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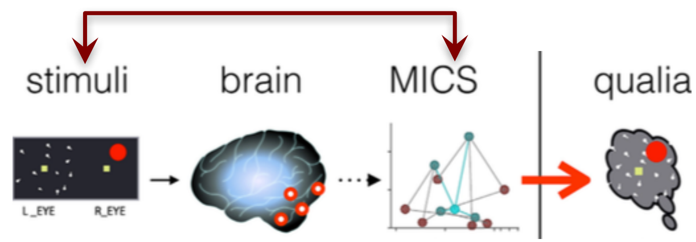
- Ideas



  - Different metrics for different simplicial complexes relative to multiple stimulus fields (visual, auditory, etc.) & computing their homology groups?
  - Homology group of MICS (which is a simplicial complex)
  - (AI) agent that can reconstruct the world from basic stimulus?

(2) Global topological features

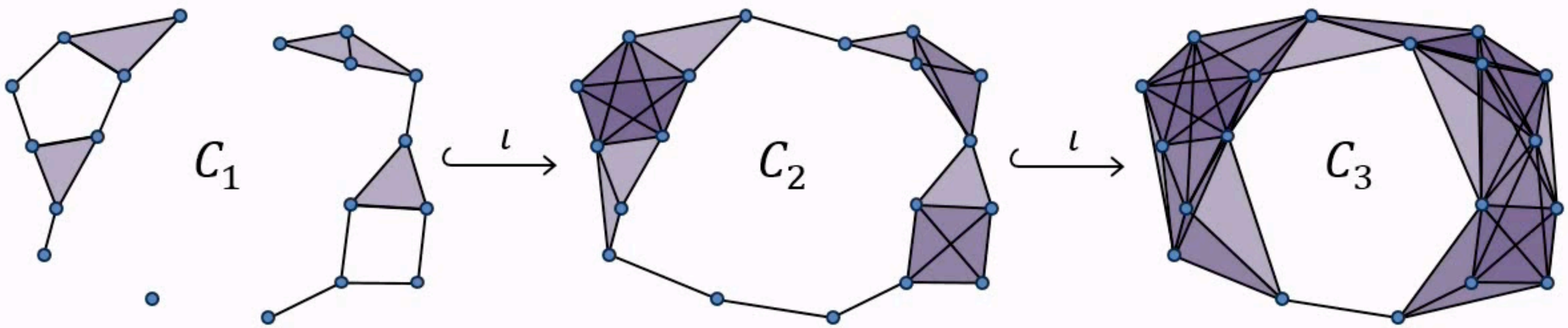
- Holes are examples of (non-metric) topological features, because they are preserved under continuous deformations of the space

- Two topologically equivalent (homeomorphic) environments can be continuously deformed into the other, and vice versa.

- Homology groups are topological invariants that can be used to distinguish topologically inequivalent spaces. In particular, the dimension of the first homology group H1 counts the number of holes. Higher order homology groups (H2, H3, …) count higher- dimensional ‘‘holes,’’ and thus place constraints on the minimum dimensionality of the space; they are all expected to vanish for flat, two-dimensional environments.

→ Homologies detect holes

Filtration:



i^{th} homology with coefficients in a field k:

$$H_i(C_1) \longrightarrow H_i(C_2) \longrightarrow H_i(C_3)$$

Persistent homology module:

graded $k[x]$-module

$$M = H_i(C_1) \oplus H_i(C_2) \oplus H_i(C_3)$$